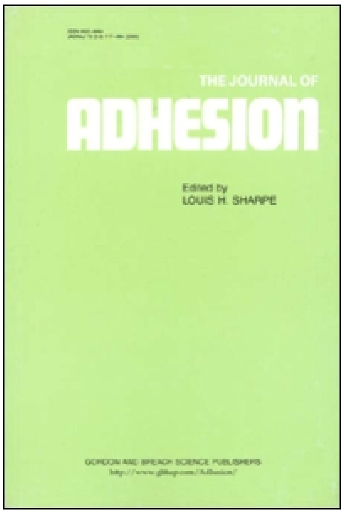

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Stresses in the Joints of Bonded Prism Assemblies

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The distribution of thermally induced shear and normal stresses at the interface between a prism and an adhesive in bonded assemblies has been obtained using the finite element method. The loading was due to a nominal uniform temperature drop of 50°C. The effects of thickness of adhesive layer, profile form at the end of the adhesive layer, angle of edge chamfer on glass and elastic moduli of glass and adhesive on these stresses have been established. The general results are also applicable to other assemblies and for stresses induced by adhesive shrinkage.

INTRODUCTION

Adhesive bonding is frequently used in optical assemblies, a typical assembly consisting of glass adherends and an optically matching adhesive. During the curing of the adhesive, shrinkage and thermal stresses are produced which are additive to stresses developed as a consequence of subsequent thermal or mechanical cycling. Such stresses determine the performance of the bonded joint as has been noticed on several occasions when debonding has occurred, local to the edge of the adhesive layer, and then grown to a sufficient extent to impair the operation of the unit or even caused fracture of glass components.

The aim of the investigation described in this paper was to establish the influence of various elastic and geometric parameters (near the exposed end of the joint) on thermal stresses. Since the analysis of shrinkage stresses is identical to that for thermal stresses, the results obtained are applicable to shrinkage stresses also. In the analysis, loading was due to a temperature

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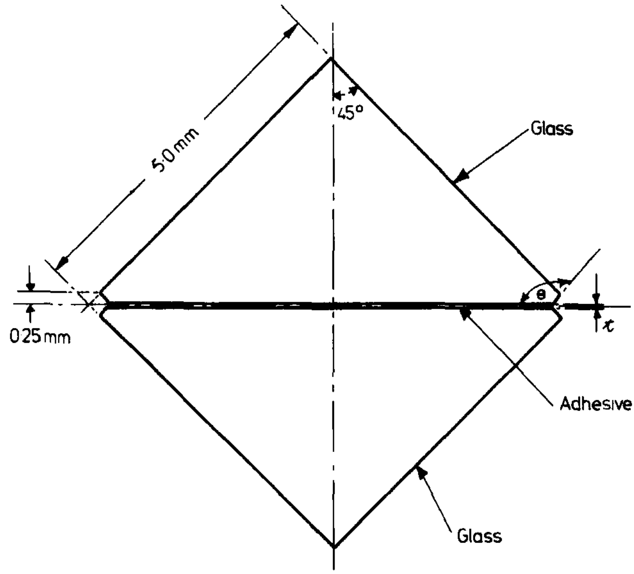


FIGURE 1 Prism assembly.

drop of 50°C and attention was restricted to the prism assembly shown in Figure 1. This Figure also shows typical dimensions and illustrates the symbols defined below. The following geometric parameters were varied:

(i) Thickness (t) of the adhesive layer.
(ii) Angle (θ) of chamfer of glass prism near the joint end.
(iii) The form of the profile at the end of the adhesive layer.

Besides geometric variations, the effect of the following elastic constants was also determined:

(iv) Young's moduli of glass (E_g) and adhesive (E_a).

The influence of parameters (ii), (iii) and (iv) has not been reported in the literature while parameter (i) has been studied for an adhesive joint between rectangular blocks or flat sheets subjected to various types of loading.[1,2,3]

NUMERICAL ANALYSIS

In this study, the analysis was based on the finite element method and was carried out within the framework of the PAFEC 70+[4] scheme. Only a very brief account of the computational parameters and procedures is given since a detailed account of the finite element method can be found elsewhere[5].

Due to the fact that the assembly shown in Figure 1 has two axes of symmetry, only one quadrant was considered. This quadrant is shown in Figure 2 and the coordinate axes are indicated in this figure. On account of

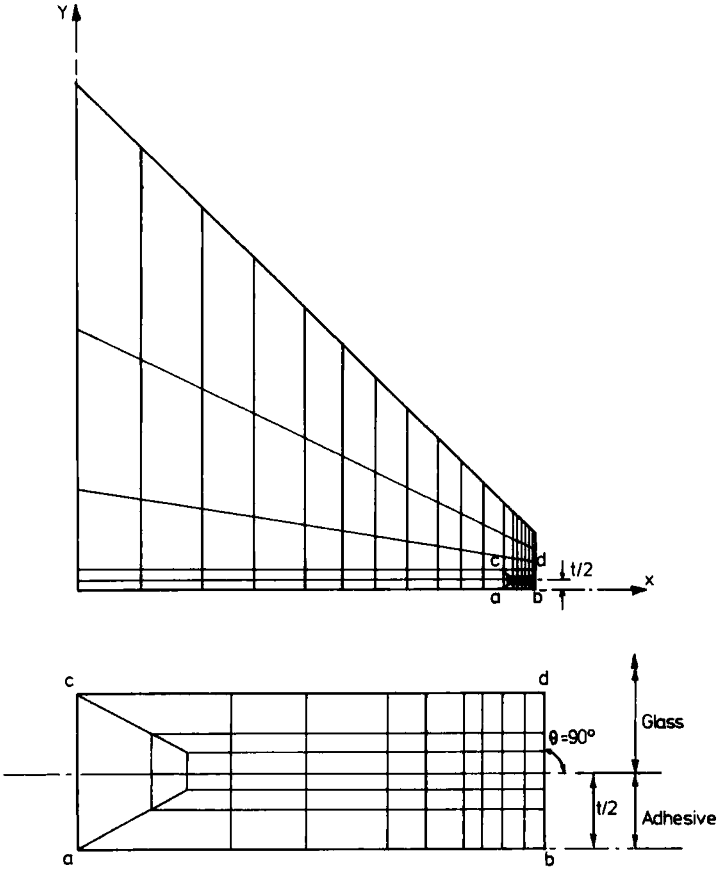


FIGURE 2 Idealization for finite element analysis.

the rapid variation in stress at the extremities of the adhesive layer, it is necessary only to look at a much reduced prism size since the stresses so calculated will be typical of any size of prism. By so doing, the number of elements required for solution will be fewer and hence computing costs less than would be the case for idealisation of the full-scale prism. The problem was treated as a plane strain problem since the dimension normal to the plane of figure was large compared to the adhesive thickness. In dealing with cases (i), (ii) and (iii), the following physical constants were used for glass and adhesive.

Glass	*Adhesive*
$E_g = 0.68 \times 10^5$ N/mm^2	$E_a = 0.117 \times 10^4$ N/mm^2
$\nu_g = 0.25$	$\nu_a = 0.35$
$\alpha_g = 0.84 \times 10^{-5}$/°C	$\alpha_a = 0.7 \times 10^{-4}$/°C

where ν and α are Poisson's ratio and coefficient of linear thermal expansion respectively. When considering the effect of variations in Young's moduli, the ν and α values remained unchanged. The value of E_a has been obtained from a report[6] issued by the Sira Institute and corresponds to the Young's modulus of the bulk material.

Figure 2 also shows the idealisation of the quadrant for carrying out the analysis. Here the angle of chamfer i is $\theta = 90°$ and the end profile of the adhesive layer is straight. When the profile was curved, a curvilinear PAFBLOCK[4] was used in the analysis. A PAFBLOCK can be thought of as a large single element comprising a block or group of elements which form a localised part of the structure. Similarly for $\theta \neq 90°$, the rectangular PAFBLOCK was replaced by a quadrilateral one. A coarser idealisation was first used and then refined gradually in regions of higher stresses, i.e., near the end of the adhesive layer (area abcd in the figure), till the stress values at a node on the interface showed satisfactory convergence when approached from glass and adhesive. This convergence was most difficult to achieve close to the end of the joint due to the presence of a singularity there, the order of which is discussed below. By using a very fine mesh in this region, the difference in stresses at the interface, when approached from the two directions, was reduced to acceptable levels. For all cases studied, the same idealisation was found to be adequate. The number of elements in this idealisation was 140 and the number of nodes approximately 475 (depending on the actual geometry).

RESULTS AND DISCUSSION

While the finite element analysis provides the values of displacement and stresses at each node, attention was paid particularly to the co-ordinate stresses σ_x, σ_y and τ_{xy}. In all cases, the stresses τ_{xy} and σ_y decreased rapidly and, in the case of the former, tended to zero directly, within a distance of approximately 1.5 to 2 times the adhesive thickness from the free end. σ_y, however, changed from tension to compression, in the same distance, before approaching zero, while σ_x levelled off at a constant value, again in the same distance. Both τ_{xy} and σ_y could cause local debonding, the first by exceeding the shear bond strength locally and the second by a peeling action resulting in tensile stresses in excess of the tensile bond strength. In both cases, crack development would be limited to the distance mentioned above because of the way in which the stresses diminish and, particularly in the case of the σ_y stress, on account of the compressive stresses inboard of the edge, which will tend to act as a buffer to crack development. The second normal stress, σ_x, is compressive in the glass and tensile in the adhesive. This could cause a transverse failure of the adhesive, but the magnitude of σ_x is well below the ultimate tensile strength of the adhesive, hence supporting the practical observation that such failures rarely occur.

In the glass, the diffusion of stresses in the y-direction is very rapid and the bulk of the glass is unstressed. This implies that the results presented below are applicable to assemblies of prisms having other shapes and different overall dimensions.

Effect of thickness of adhesive

This was investigated for the case $\theta = 90°$ (straight chamfer) and straight adhesive layer and profile. Three thicknesses were studied, 0.05 mm, 0.03 mm and 0.02 mm. The stress distributions in glass and adhesive are shown in Figure 3. There is obviously a singularity at the end of the adhesive layer and, near the singularity, the stress values are likely to differ for glass and adhesive in any numerical integration. In Figure 4, the same stress distributions are plotted as a function of x'/t where x' is the distance from the free end of the interface. The τ_{xy}-distributions show a very close agreement for all thicknesses whereas, near the end of the layer, the distributions for σ_x and σ_y show some variations. This appears to be the effect of computation as σ_x and σ_y gradients in the neighbourhood of the singularity are higher than that for τ_{xy}. This figure leads to the conclusion that stresses at the end of the bonded joint are independent of thickness of adhesive layer as suggested by Harrison and Harrison[2] and Dukes and Bryant[3].

Effect of chamfer

The shear stress distribution at the interface for three cases of chamfer, i.e., $\theta = 70°$, $90°$ and $110°$, are shown in Figure 5, and it can be seen that there are no significant differences between them. In these cases other parameters were unchanged. The normal stress distributions at the interface in the adhesive are shown in Figure 6. Again the differences are very small and, if anything, the reentrant chamfer ($\theta = 110°$) is marginally worse.

Effect of end profile of adhesive layer

Three shapes were considered, namely straight, shallow parabolic and deep parabolic. The shear stress distributions (Figure 7) show a marked decrease near the end of the adhesive layer as the depth of the parabolic form increases. In fact there is no longer a singularity if the wedge angle of the profile, defined as β in Figure 7, is less than about 50°.

From the theoretical analysis[7] it can be seen that the nature of the singularity near the end of the adhesive layer is influenced by:

(i) Poisson's ratio of the adhesive

(ii) Angle (β) between the adhesive edge and the interface.

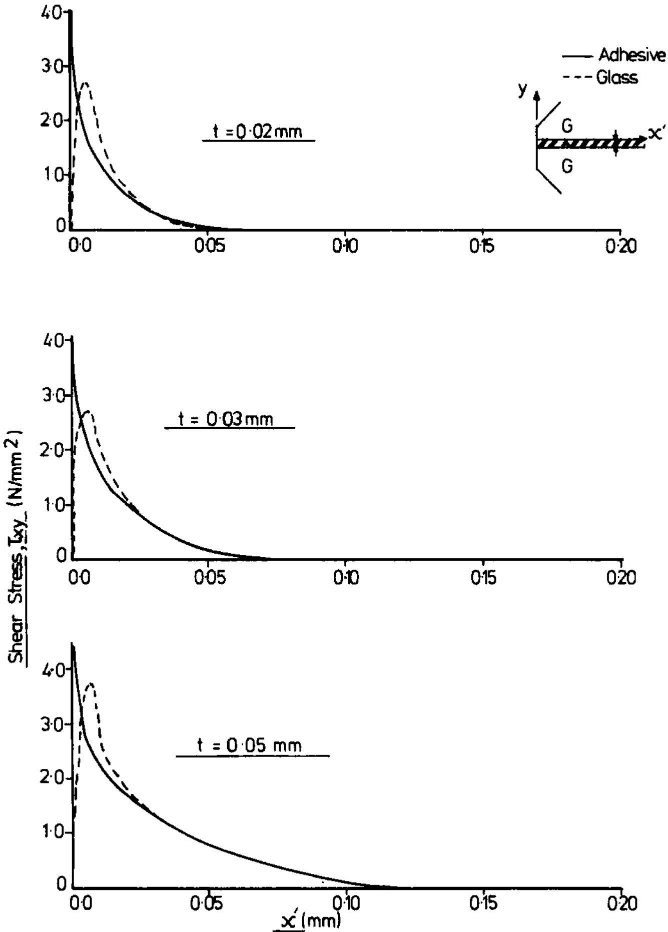


FIGURE 3 Shear stress distributions for various thicknesses of adhesive layer—straight chamfer, straight adhesive layer end profile.

The singularity in stresses is of the type $r^{-\lambda}$ where r is the distance from the corner and λ is a real constant. It attains its maximum value for $\nu = 0.5$ and $\beta = 180°$. For $\beta = 90°$ (straight profile) and $\nu = 0.35$, the value of $\lambda \simeq 0.32$. Thus the stress variation along the interface is of the following type:–

$$\text{stress} \propto (x')^{-0.32}$$

This relationship appears to hold very near the edge ($0 < x' \leqslant 0.01$ mm).

The singularity implies that a microcrack can be produced by thermal loading or by shrinkage since, as mentioned previously, the analyses for

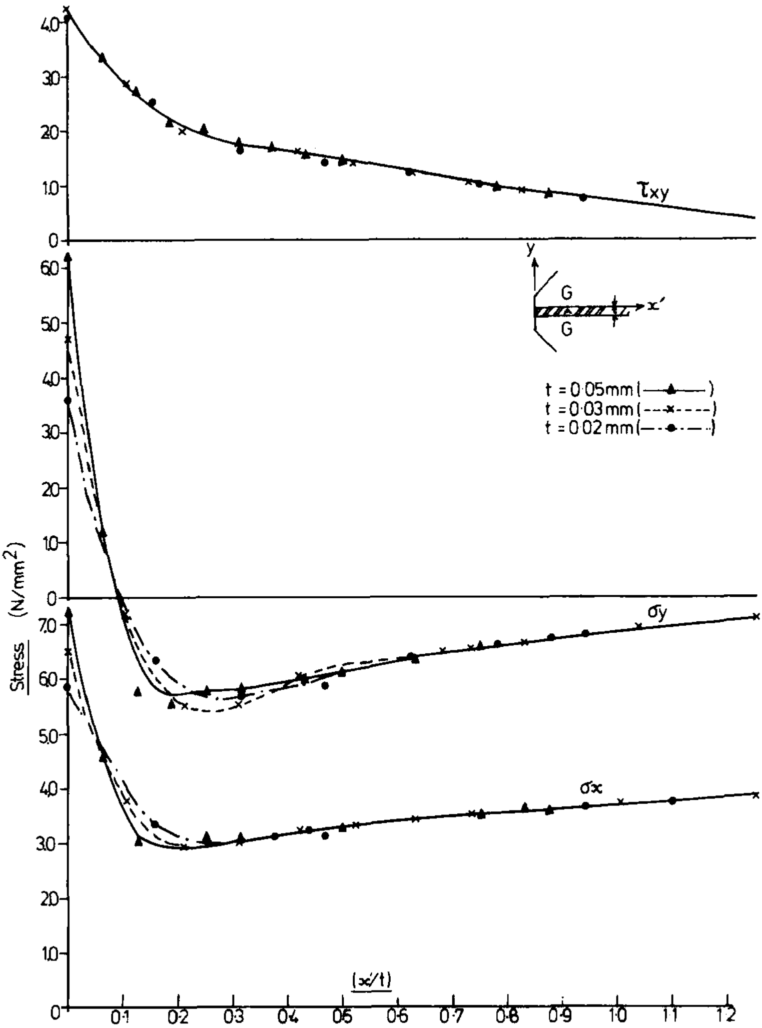


FIGURE 4 Comparisons of stress distributions at interface in terms of $(x'(t)$.

thermal and shrinkage conditions are identical. However, the viscoelastic nature of the adhesive before it fully cures, or local yielding, can modify or eliminate the singularity.

One way of reducing λ is to decrease β, that is by producing a deeper profile at the end of the adhesive layer. For $\beta < 50°$, $\lambda \leqslant 0$, and hence the singularity does not exist. For an adhesive having a high Poisson's ratio ($\nu \simeq 0.5$) this happens when $\beta < 45°$. From the numerical analysis presented

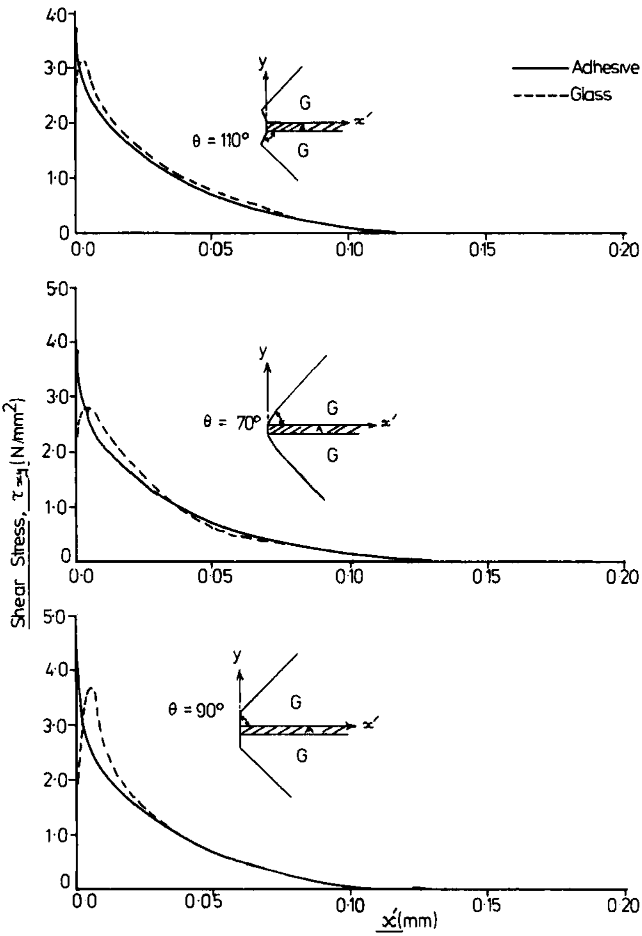


FIGURE 5 Shear stress distribution at interface for various glass edge chamfers—straight adhesive layer end profile and adhesive thickness = 0.05 mm.

in this paper, the same conclusion can be drawn. However if $\beta > 90°$, that is, for a slight excess of adhesive, λ increases and hence the stresses rise more sharply. Therefore this situation should be avoided in practice.

Figure 8 shows the behaviour of the normal stresses as the adhesive end profile changes. For clarity and in order to highlight the advantages of a $\beta \leqslant 90°$, only stresses for straight and deep profile cases are plotted. It is evident that the presence of a deep profile is helpful in reducing all stresses.

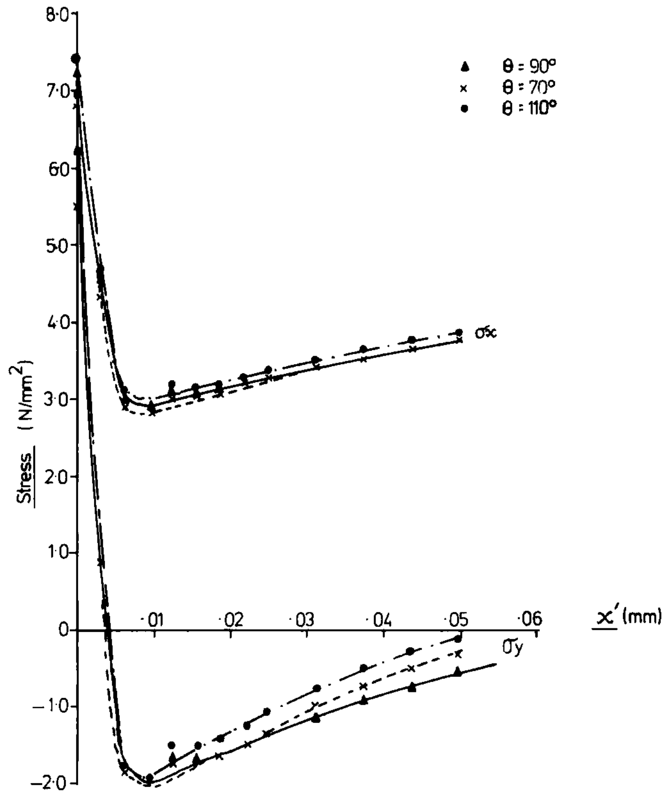


FIGURE 6 Effect of chamfer on normal stresses in adhesive at the interface.

Effect of Young's moduli of both glass and adhesive

The next variables to be considered were Young's moduli of both glass and adhesive. According to Harrison and Harrison[2], whose analysis considered the adherend to be rigid ($E_g = \infty$) stresses are proportional to the Young's modulus of the adhesive (E_a). In the present study, three representative values of E_g have been considered:

$$E_g = 0.68 \times 10^5 \text{ N/mm}^2,\ 1.1 \times 10^5 \text{ N/mm}^2 \text{ and } 0.68 \times 10^9 \text{ N/mm}^2,$$

the latter value being for comparison only. Similarly for E_a, two representative values have been considered:

$$E_a = 0.117 \times 10^4 \text{ N/mm}^2 \text{ (soft adhesive) and}$$

$$E_a = 0.5 \times 10^4 \text{ N/mm}^2 \text{ (hard epoxy adhesive).}$$

These values cover almost the entire range of glasses and adhesives used in optical assemblies. In order clearly to demonstrate the effect of moduli

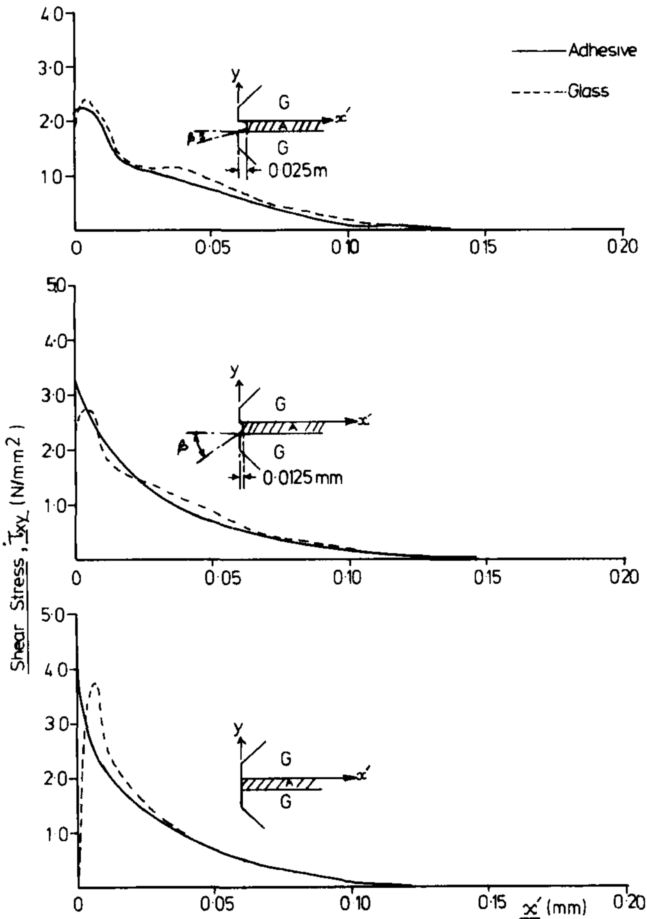


FIGURE 7 Shear stress distribution for various adhesive layer end profiles—straight chamfer, adhesive thickness = 0.05 mm.

variations, the case of a deep profile at the end of the adhesive layer and straight chamfer of glass edge has been examined. As seen above, the use of a deep profile eliminates the uncertainty near the free end arising due to the singularity. The comparison for τ_{xy}-distributions for four cases considered is shown in Figure 9. It is obvious that whereas the effect of change of glass modulus is very small, the effect of the change of adhesive modulus is very much greater. In fact, an adhesive with a low stiffness is preferable since this results in smaller stresses at the interface.

The behaviour of normal stresses with changes in E_a and E_g, although not plotted, shows a similar pattern. The normal stress σ_x levels off to a

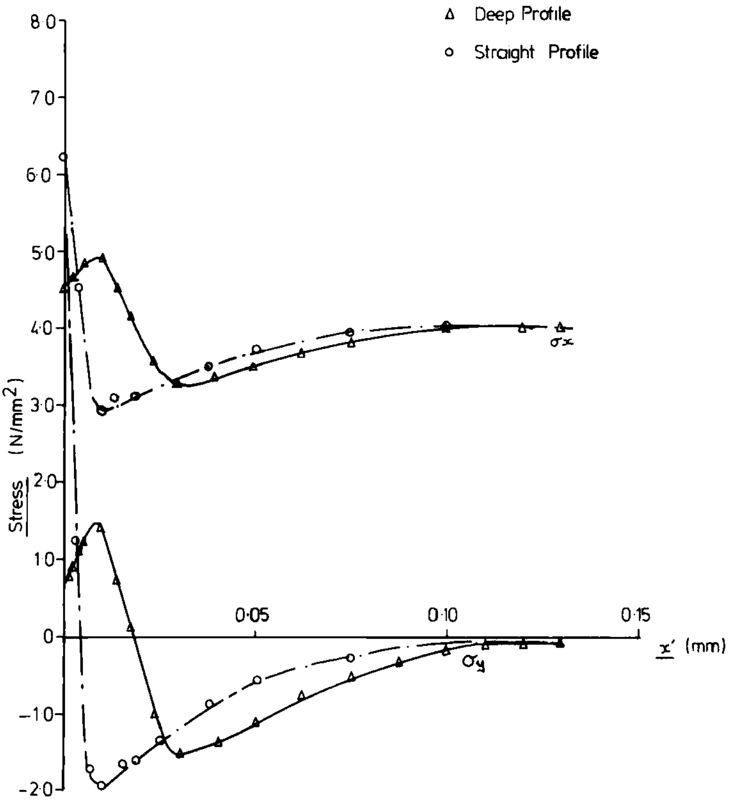
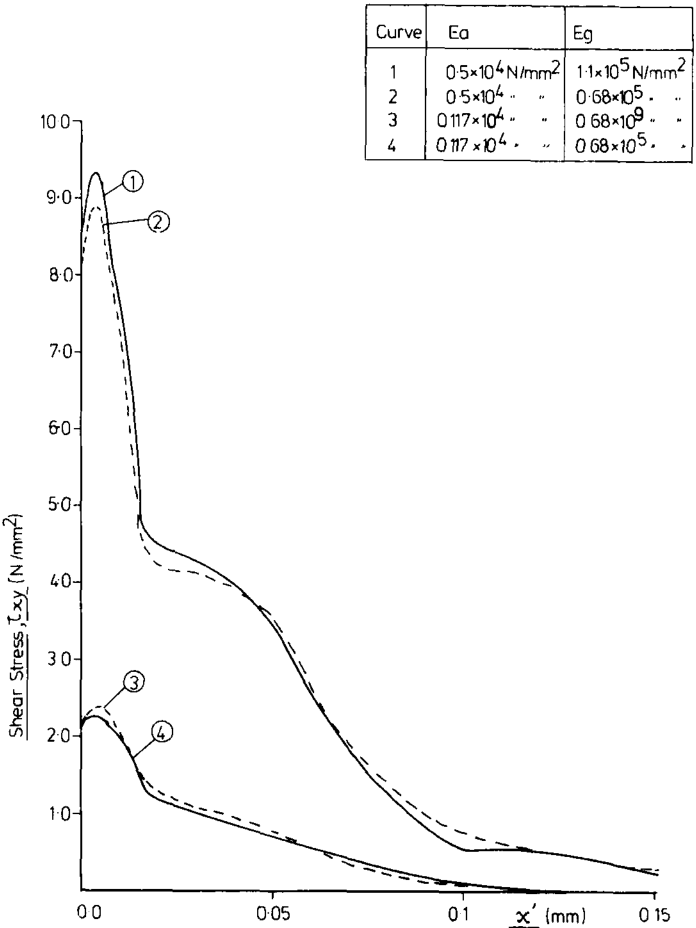


FIGURE 8 Effect of adhesive layer end profile on normal stresses in adhesive at the interface.

constant value within 0.15 mm from the edge. This constant value is proportional to E_a. Other features, such as the change from tensile to compressive σ_y within about $t/3$, are also preserved when E_a changes.

It should be noted from Figure 9 that the stresses, obtained for the systems having the higher E_a values, lie in the region of the measured shear strengths for such adhesives and hence the possibilities of failure when these adhesives are used is greater than that for adhesives having low E_a values.

The strong dependence of stresses on Young's modulus (E_a) of the adhesive raises the question of what is the influence of temperature on the bonded joints? Certainly environmental checks are made on units incorporating bonded joints, in which temperature excursions from +70°C to −70°C are fairly common, with the possibility of exceeding these temperatures on occasions. Service requirements can also produce a fairly wide range of temperatures. There is evidence to suggest[8] that, relative to room temperature



Curve	Ea	Eg
1	$0{\cdot}5\times10^4$ N/mm^2	$1{\cdot}1\times10^5$ N/mm^2
2	$0{\cdot}5\times10^4$ " "	$0{\cdot}68\times10^5$ " "
3	$0{\cdot}117\times10^4$ " "	$0{\cdot}68\times10^9$ " "
4	$0{\cdot}117\times10^4$ " "	$0{\cdot}68\times10^5$ " "

FIGURE 9 Comparison of τ_{xy}-distributions in adhesive for different values of E_a and E_g.

values, Young's Modulus decreases as temperature increases and increases with a reduction in temperature. This suggests that there would be a greater incidence of failures at lower temperatures than at higher temperatures and this appears to be the case observed in practice. Thus, in choosing an adhesive, consideration should not only be given to the room temperature characteristics, but also to how these properties may vary with temperature fluctuations over a practical range.

In the calculations reported here, the expansion coefficients of the adhesive and glass have remained constant, whilst the other parameters have been varied. However, it is considered worth making the point that the benefit in a reduced Young's modulus comes only by the expansion coefficient remaining relatively unchanged or possibly reducing in order to ensure an overall reduction in the product, $E_a(\alpha_a-\alpha_g)$. If $(\alpha_a-\alpha_g)$ increased as E_a decreased to maintain a constant $E_a(\alpha_a-\alpha_g)$, there would be no stress change. Hence the variation of expansion coefficients of both glass and adhesive over the selected temperature range is also important.

Extension of results to other problems

It has been observed for all cases examined that the shear stress τ_{xy} in the glass decreases very rapidly as the distance from the interface increases. The attenuation of the normal stress σ_y is slower. However, both stresses become very small within a distance equal to the adhesive layer thickness from the interface for all cases. The bulk of the prism remains unstressed. This fact makes the above results applicable to a wider range of problems, e.g., glass plate or rectangular block assemblies, the only restriction being that glass thickness should be much greater than that of the adhesive.

Although the results described previously are relevant only to the specific glass-adhesive systems studied here, knowledge gained from these systems can add to the general understanding of the edge effects in other types of problems involving bonded joints, such as laminates of composite materials. The results reported here are in qualitative agreement with a finite element investigation of thermally induced stresses in fibre composite laminates with different ply orientations[9], where the nature of edge singularity has been explored using an advanced sparse matrix computational technique. In that investigation only the ply orientations were varied. The general nature of the results obtained in this paper is also applicable to shrinkage problems since the governing equations for thermal and shrinkage stresses are identical.

CONCLUSIONS

i) In all cases investigated, maximum stresses (shear as well as normal) along the interface, occur at the free end of the adhesive layer. These stresses decrease to negligible values within a distance approximately equal to twice the thickness of the adhesive layer. For particular combinations of joint parameters, stresses sufficiently high to cause failure can be developed.

ii) The most significant parameter in terms of thermal stresses generated at the glass interface is the product of Young's modulus of the adhesive and the difference between expansion coefficients of glass and adhesive—the larger the value of this product, the higher will be the stresses. The contribution to changes in overall stresses made by variations in Young's modulus of glass is small.

iii) The most significant parameter in terms of shrinkage stresses generated at the interface is simply Young's modulus of the adhesive.

iv) As the stresses are a function of the product of the difference of expansion coefficients and Young's modulus of the adhesive, at the test temperature, the room temperature properties do not provide a valid basis for comparison of different adhesives. In order properly to assess the merits of various adhesives it is necessary to know the variation of these physical properties with temperature.

v) The form of the profile at the end of the adhesive layer has an important effect on stresses. All stresses decrease as the profile becomes deeper. An overflow of adhesive should be avoided.

vi) The effect of adhesive layer thickness on the stresses can be treated as an effect of scaling since they show identical distributions in terms of (x'/t) for all thicknesses.

vii) The chamfer at the edge of the prism has very little effect on stresses.

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REFERENCES

1. B. J. Thomas and H. W. McKenzie, *Analysis of Stresses in Adhesive Layers of Multicomponent Prism Assemblies*, Pilkington Bros. R & D Report SC/76/19.
2. N. L. Harrison and W. J. Harrison, *J. Adhesion* **3**, 195 (1972).
3. W. A. Dukes and R. W. Bryant, *J. Adhesion* **1**, 48 (1969).
4. R. D. Henshall, **PAFEC 70+** Manual (plus extensions to 1974) **PAFEC** Ltd., Strelley Hall, Strelley, Nottingham, NG8 JPE, England.
5. O. C. Zienkiewicz, *The Finite element method in Engineering Science* (McGraw Hill, London, 1971).
6. P. G. Hunt, S. Martin and J. M. B. Webber, *Optical cements—A Laboratory Assessment (II)*, SIRA Institute Report on Contract No. A/CTS/SI/5687/CB/INSTS/1(2).
7. P. W. M. Brighton, *The Thermal Contraction of Adhesive Layers in Optical Components*, R & D Report, Pilkington Bros. Ltd.,
8. L. Yu, Kurts, M. A. Azar'eva, G. F. Pishchik and E. N. Prokof'ev, *Sov. J. Opt. Tech.*, **42**, 464 (1975).
9. A. S. D. Wang and F. W. Crossman, *J. Comp. Matls.*, **11**, 92 (1977).